Fermi National Accelerator Laboratory

Considerations of Using Siberian Snakes for Very Strong and Very Weak Resonances*

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We use the spinor formalism where the spinor operator

$$e^{\frac{i}{2}\phi(n\bullet\vec{\sigma})} \equiv \cos\frac{\phi}{2} + i(n\bullet\vec{\sigma})\sin\frac{\phi}{2} \tag{1}$$

corresponds to a precession of angle ϕ about the axis $\hat{n}$. The operator M corresponding to a succession of precessions is just the produce of a series of operators of the type given in Eq. (1). The combined single angle ϕ and axis $\hat{n}$ of the succession of precessions are then given by

$$\begin{cases} \cos\frac{\phi}{2} = \frac{1}{2}\,\mathrm{Tr}(M) \\[2ex] n = \dfrac{1}{2i\sin\frac{\phi}{2}}\,\mathrm{Tr}(\vec{\sigma}M) \end{cases} \tag{2}$$

Going through a resonance of frequency κ and strength $\epsilon = \epsilon_R + i\epsilon_I \equiv |\epsilon|e^{i\lambda}$ the operator is

$$e^{\frac{i}{2}\theta(\vec{w}\bullet\vec{\sigma})} = e^{\frac{i}{2}\theta w\sigma_w} \tag{3}$$

where θ is the angle around the ring,

$$\vec{w}\bullet\vec{\sigma} = \epsilon_R\sigma_x - \epsilon_I\sigma_y - \delta\sigma_z = \begin{pmatrix} -\delta & \epsilon \\ \epsilon* & \delta \end{pmatrix} \tag{4}$$

and $\delta = \gamma G - \kappa$ = frequency deviation from resonance

$$w \equiv |\vec{w}| = \sqrt{\delta^2 + |\epsilon|^2}\,, \qquad \sigma_w \equiv w\bullet\vec{\sigma}\,.$$

A longitudinal snake (type 1) has the operator

$$e^{\frac{i}{2}\pi\sigma_y} = i\sigma_y \tag{5}$$

and a transverse snake (type 2) has the operator

$$e^{\frac{i}{2}\pi\sigma_x} = i\sigma_x \ . \tag{6}$$

With a single transverse snake the one-revolution operator is

$$M = e^{\frac{i}{2}\pi w\sigma_w}(i\sigma_x)e^{\frac{i}{2}\pi w\sigma_w} \tag{7}$$

and the spin tune ν_p is given by

$$\cos \pi\nu_p = \tfrac{1}{2} \ \mathrm{Tr}\,(M) = -\frac{\epsilon_R}{w}\sin \pi w \tag{8}$$

When $\epsilon_R = 0$, $\nu_p = 1/2$ and there is no resonance. Resonance appears only when $\nu_p = 0$ or 1 which can occur only at

$$\delta = 0, \qquad \frac{\epsilon_R}{|\epsilon|} = \cos \lambda = \pm 1 \quad \text{and} \quad |\epsilon| = \pm \tfrac{1}{2} \tag{9}$$

Thus the resonance strength that can be suppressed by one snake is $-\tfrac{1}{2} < |\epsilon| < \tfrac{1}{2}$ or since $|\epsilon|$ must be positive, $0 < |\epsilon| < \tfrac{1}{2}$. With a pair of snakes,

$$M = (i\sigma_y) \ e^{\frac{i}{2}\pi w\sigma_w} \ (i\sigma_x) \ e^{\frac{i}{2}\pi w\sigma_w} \tag{10}$$

and

$$\cos \pi\nu_p = \tfrac{1}{2} \ \mathrm{Tr}\,(M) = -2 \ \frac{\epsilon_R\epsilon_I}{w^2} \ \sin^2 \frac{\pi w}{2} \ . \tag{11}$$

The range of strength that can be suppressed becomes $0 < |\epsilon| < 1$.

A. Very Strong Resonances

For SSC[1], $|\epsilon|$ can get as large as 8 and we need N pairs of snakes evenly distributed around the ring. The spin tune equation (11) then becomes

$$\cos \pi\frac{\nu_p}{N} = -2 \frac{\epsilon_R \epsilon_I}{\omega^2} \sin^2 \frac{\pi\omega}{2N} \tag{12}$$

The right-hand-side is largest when $\delta = 0$ and

$$2 \frac{\epsilon_R \epsilon_I}{|\epsilon|^2} = 2 \sin \lambda \cos \lambda = \sin 2\lambda = -1 \ .$$

This gives

$$\cos \pi\frac{\nu_p}{N} = \sin^2 \frac{\pi|\epsilon|}{2N} \ . \tag{13}$$

R. Ruth[2] imposed the condition that for all resonances the tune ν_p should lie between successive integers and concluded that N should be approximately proportional to $|\epsilon|^2_{max}$. This leads to a very large number of snakes. One can, however, get by with the less luxurious condition that all values of $2\nu_p/N$ should lie between 1 and 0, namely $\cos \pi\nu_p$ should lie between 0 and 1. This condition gives N approximately proportional to $|\epsilon|_{max}$, a much smaller number.

To understand this, we plot ν_p against $|\epsilon|$ for N = 50 (approximately proportional to $|\epsilon|^2_{max}$) and for N = 8 (approx. proportional to $|\epsilon|_{max}$) in Fig 1. With N = 50, resonance strengths $|\epsilon|$ from 0 to 8 gives tunes ν_p lying in the range of 25 to 24, hence there is no danger of landing on integer values. With N = 8, the same $|\epsilon|$ range of 0 to 8 give ν_p values ranging from 4 to 0, thus running the risk of having to cross one or more of the integer values 3, 2 and 1. This led to the requirement imposed by Ruth. However, Eq. (13) is written in the reference frame rotating at the resonance frequency. In the laboratory frame, the right-hand-side is multiplied by a phase factor which depends on the local orbit phase and which effectively makes the depolarizing precessions from one snake pair to the next incoherent. Thus, the spin-time factor ν_p/N in Eqs. (12) and (13) should be replaced by just ν_p. This, then leads to a linear dependence between N and $|\epsilon|_{max}$.

B. Very Weak Resonances

For low energy machines with very weak resonances, one snake is more than enough. The difficulty here is that the transverse orbit excursions in the dipoles of a fully excited snake are rather large at low energies making the required aperture of these dipoles excessively large. One is therefore lead to considering[3] partially excited snakes for which the precession angle is less than π. For such a partial transverse snake, we write the precession angle as $\pi(1-2a)$ with $a<1/2$, and the spinor operator as

$$e^{\frac{i}{2}\pi(1-2a)\sigma_x} = e^{-\frac{i}{2}\pi a \sigma_x}(i\sigma_x) \, e^{-\frac{i}{2}\pi a \sigma_x} \tag{14}$$

The one-revolution operator can then be written in the physically revealing form

$$\begin{aligned}
M &= e^{\frac{i}{2}\pi w \sigma_w} \, e^{-\frac{i}{2}\pi a \sigma_x}(i\sigma_x) \, e^{-\frac{i}{2}\pi a \sigma_x} \, e^{\frac{i}{2}\pi w \sigma_w} \\
&\equiv e^{\frac{i}{2}\pi \mu \sigma_\mu}(i\sigma_x) \, e^{\frac{i}{2}\pi \mu \sigma_\mu}
\end{aligned} \tag{15}$$

Comparing Eq. (15) with Eq. (7), we see that the effect of a is merely to modify $\vec{w}$ to $\vec{\mu}$. The system is equivalent to a full snake but with a stronger resonance. The equivalent resonance strength $|\epsilon_\mu|$ of μ is larger than $|\epsilon|$, but as long as $|\epsilon_\mu| < 1/2$ the resonance is effectively eliminated. We can now get the spin tune ν_p by

$$\cos \pi \nu_p = \frac{1}{2}\, \text{Tr}(M) = -\frac{\epsilon_R}{w}\, \sin \pi w \cos \pi a + \cos \pi w \sin \pi a \tag{16}$$

Here, again, resonance can occur only with $\delta = 0$ and $\cos \lambda = \pm 1$ for which

$$\cos \pi \nu_p = \begin{cases} -\sin \pi(|\epsilon| - a) & \text{at } \cos\lambda = 1 \\ \sin \pi(|\epsilon| + a) & \text{at } \cos\lambda = -1 \ . \end{cases} \tag{17}$$

Resonances appears at $\nu_p = 0$ and 1 when

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$$|\epsilon| = \begin{cases} \frac{1}{2} - a, & -\frac{1}{2} + a \qquad (\nu_p = 0) \\ \frac{1}{2} + a, & -\frac{1}{2} - a \qquad (\nu_p = 1) \end{cases} \qquad (18)$$

These 4 lines are plotted in Fig. 2. The shaded area corresponds to the allowed spin tune value $0 < \nu_p < 1$. Thus, for weak resonances with small $|\epsilon|$, a can be almost as large as $(1/2) - |\epsilon|$ or the required snake precession angle can be almost as small as

$$\pi(1-2a) = 2\pi|\epsilon| \qquad (19)$$

As the resonances get stronger at higher energies, the required snake precession angle increases until when $|\epsilon|$ approaches 1/2 the full snake with π precession angle is, then, needed. Although so far we do not have a snake with continuously adjustable precession angle, there is no doubt that such a snake can be invented and designed. The transverse orbit excursions in these snake dipoles should be approximately proportional to the precession angle, significantly reduced to acceptable values at low energies.

Physically the actions of the multiple and the partial snakes for strong and weak resonances are easy to understand. Figuratively, the accumulation of depolarizing precessions at a resonance can be thought of as the resonant excitation of an oscillator. One snake in the ring jumps the oscillator phase by π so that the resonant force alternately excite and damp the oscillator in successive turns. For very strong resonant forces the phase shifts must occur closer together than one whole circumferance, otherwise the excitation in between phase shifts may get too big. For very weak resonant forces, each phase shift needs only be π/k, say. even though the oscillation is excited over k passages (or k revolutions, if only one snake in ring), the total excitation is still tolerably small. Over the subsequent k passages, the oscillation will be damped back.

With these extended modes of operation, Siberian snakes can be applied to synchrotrons or storage rings from a few GeV up to well beyond the energy of the SSC. Of course, all these analyses should be verified by numerical tracking studies.

References

1. E.D. Courant, S.Y. Lee, and S. Tepikian, "Calculation of Spin Depolarizing Resonance Strength", AIP Conf. Proc. No.145, Polarized Beams at SSC, p.174 (1985).

2. R.D. Ruth, "Report of the Working Group for Polarization in the SSC Main Ring", ibid p.62 (1985).

3. First Proposed by T. Roser.

Figure 1. Spin tune ν_p as given by Eq. (13) plotted against resonance strength $|\epsilon|$ for sin 2λ = -1 and for two different numbers, N, of snake pairs. For N = 50, values of ν_p corresponding to $|\epsilon|$ ranging from 0 to 8, all lie in between integers 24 and 25. For N = 8, they lie in between 0 and 4. Eq. (13) should, however, be modified.

Figure 2. The a vs. $|\epsilon|$ plot for the partial snake giving the 4 limiting lines corresponding to ν_p = 0 and 1. The shaded area contains allowed ν_p values. Operating on the "partial snake line" shown will not lead to excessively large orbit excursions even at low energies.

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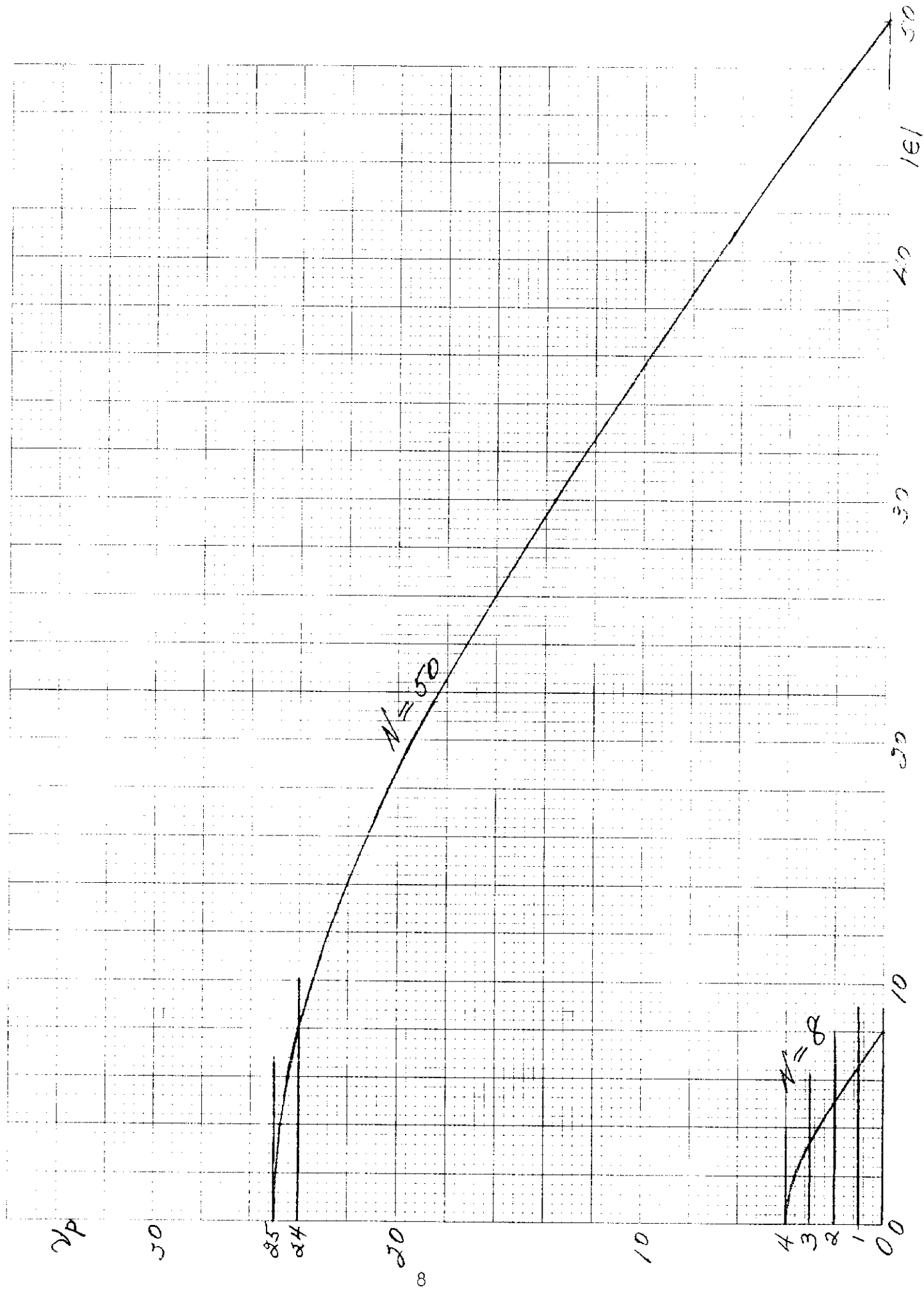
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ENGINEERING NOTE

SECTION | PROJECT | SERIAL-CATEGORY | PAGE

SUBJECT

NAME

DATE | REVISION DATE

a

$|\epsilon| = -\frac{1}{2} + a \quad (\nu_p = 0)$

PARTIAL SNAKE LINE

FULL SNAKE LINE $(a=0)$

$|\epsilon| = \frac{1}{2} + a \quad (\nu_p = 1)$

$\frac{1}{2}$

$-\frac{1}{2}$

0

$\frac{1}{2}$

$|\epsilon|$

$|\epsilon| = \frac{1}{2} - a \quad (\nu_p = 0)$

$-\frac{1}{2}$

$|\epsilon| = -\frac{1}{2} - a \quad (\nu_p = 1)$